

Spring 2020 Math 245 Exam 1

Please read the following directions:

Please write legibly, with plenty of white space. Please fill out the box above as legibly as possible. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 10:00 and will end at 10:50; pace yourself accordingly. Please remain quiet to ensure a good test environment for others. Good luck!

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. even

b. floor

c. Double Negation Theorem

Problem 2. Carefully define the following terms:

a. Trivial Proof Theorem

b. Contrapositive Proof Theorem

c. converse

Problem 3. Let a, b, c be integers, with $a|b$ and $a|c$. Prove that $a|(b + c)$.

Problem 4. Let $m, n \in \mathbb{Z}$ with $m \geq n \geq 0$. Prove that $\binom{m}{n} = \binom{m}{m-n}$.

Problem 5. Use truth tables to prove that $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$.

Problem 6. Let $x \in \mathbb{R}$. Prove that if 6 is irrational, then x is irrational.

Problem 7. Prove or disprove: $\forall x \in \mathbb{Z}, x + 1 > x$.

Problem 8. Let p, q, r, s be propositions. Simplify $(p \rightarrow q) \rightarrow (r \rightarrow s)$ to use only \vee, \wedge, \neg where only basic propositions are negated.

Problem 9. State Modus Ponens and prove it using other theorems (without truth tables).

Problem 10. Prove or disprove: $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \exists z \in \mathbb{R}, y^2 \leq x^2 < z^2$.